The Polyhedral Model Beyond Loops

Recursion Optimization and Parallelization Through Polyhedral Modeling

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Outline

1. Introduction
2. Proposed Solution: From Recursive Functions to Optimized Loops
3. Case Studies
4. Conclusion and Perspectives

Motivation

There may be a huge gap between:

- the statements in a program source code
- the instructions actually performed by a given processor architecture

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- the statements in a program source code
- the instructions actually performed by a given processor architecture

Efficient optimizations may be applied as soon as the actual runtime behavior has been discovered

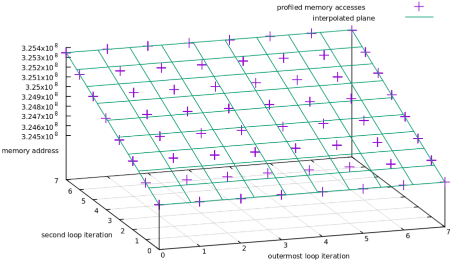
- dedicated to specific control structures & memory access patterns

Inspiration

Apollo

- Captures a polyhedral behavior of loops at runtime
- Applies the polyhedral model

Memory Accesses Behavior at Runtime from statically non-polyhedral loops!



Inspiration

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We apply the Apollo Approach for codes that are originally not loops!
=> **recursions**

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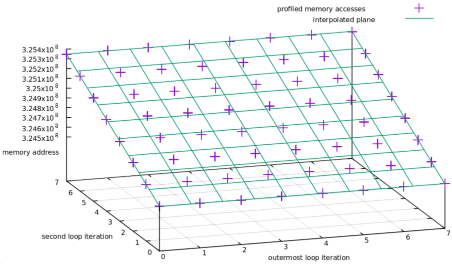

Objectives

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1. optimizing recursive functions through transformation into affine loops
2. extending the scope of polyhedral optimizations to cover recursive functions

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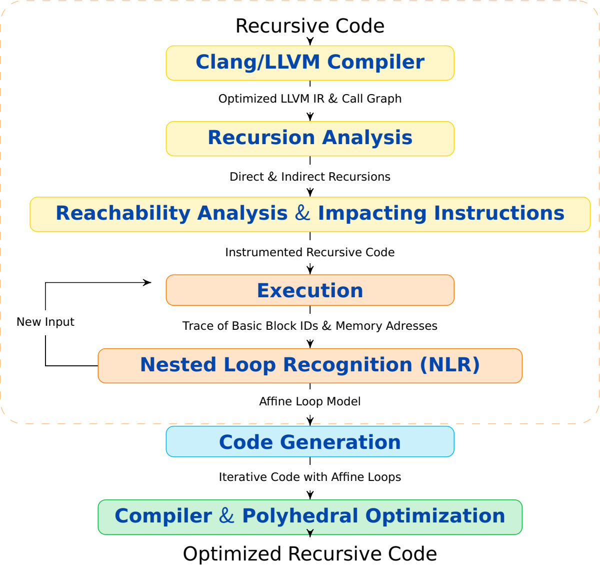
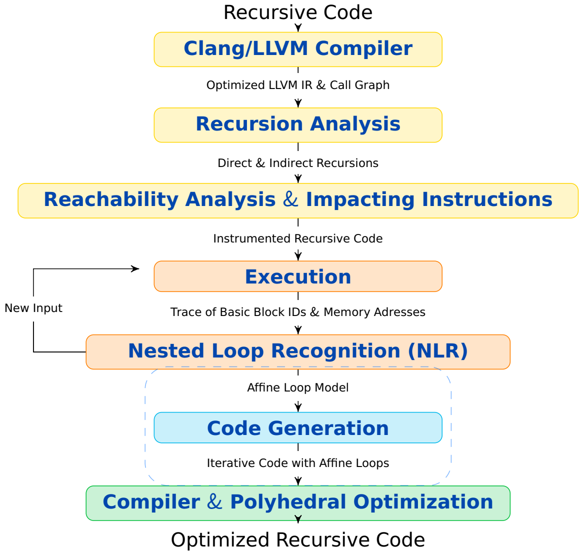
Implementation



The implementation consists of 3 main steps:

1. Recursive Control and Memory Behavior Analysis

Implementation



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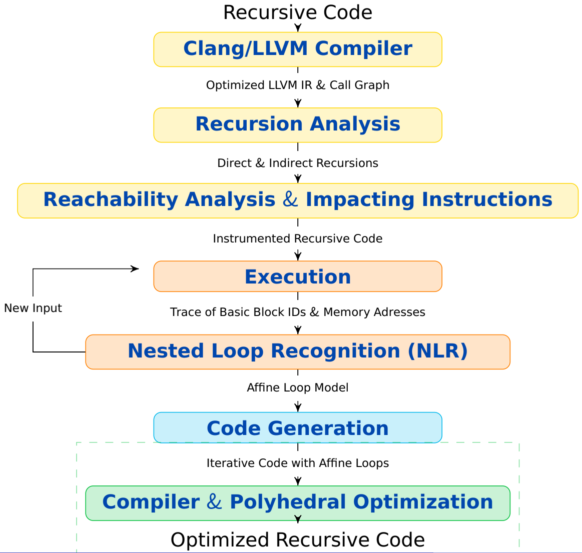
Recursive Code

Clang/LLVM Compiler

Optimized LLVM IR & Call Graph

Recursion Analysis

Direct & Indirect Recursions

Reachability Analysis & Impacting Instructions

Instrumented Recursive Code

Execution

New Input

Trace of Basic Block IDs & Memory Adresses

Nested Loop Recognition (NLR)

Affine Loop Model

Code Generation

Iterative Code with Affine Loops

Compiler & Polyhedral Optimization

Optimized Recursive Code

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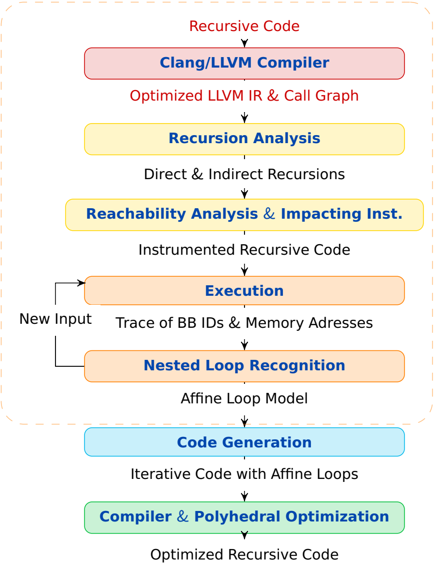
3. Polyhedral Optimizations

Recursive Control and Memory Behavior Analysis



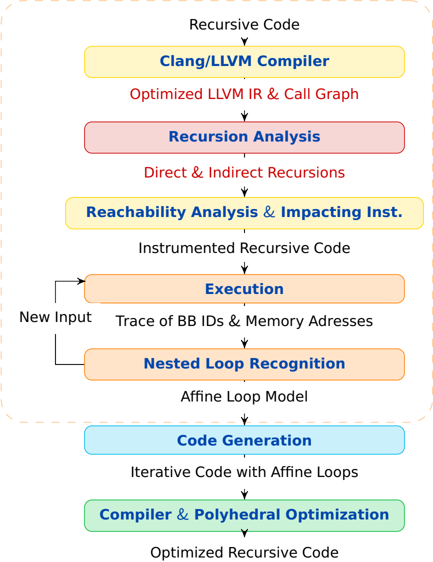
Input: recursive code

Apply classical LLVM optimization passes

- promote memory to register
- simplify CFG
- dead code elimination

Output: optimized LLVM IR & call graph

Recursive Control and Memory Behavior Analysis



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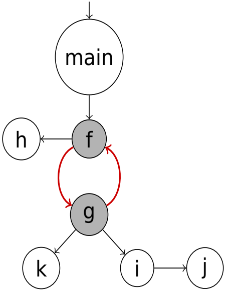
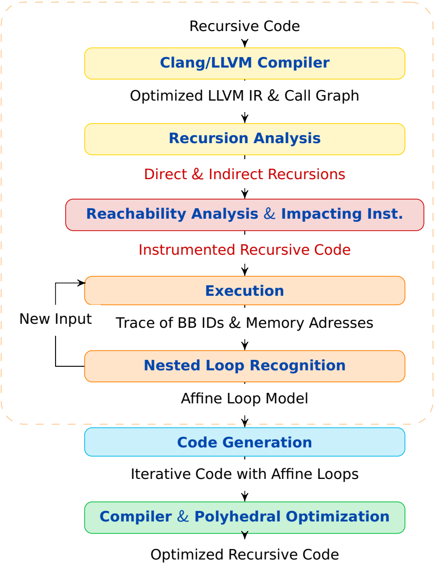


Output: direct & indirect recursions

Recursive Control and Memory Behavior Analysis



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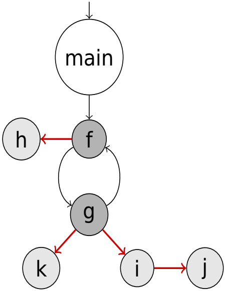
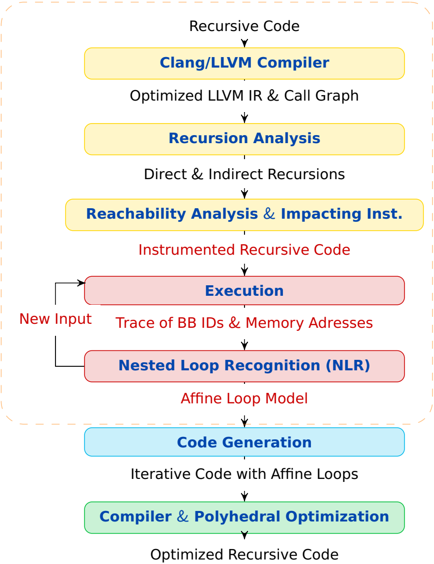
Input: direct & indirect recursions



Output: instrumented recursive code

Recursive Control and Memory Behavior Analysis



Input: Trace of the program execution : Basic Block IDs & Memory Addresses

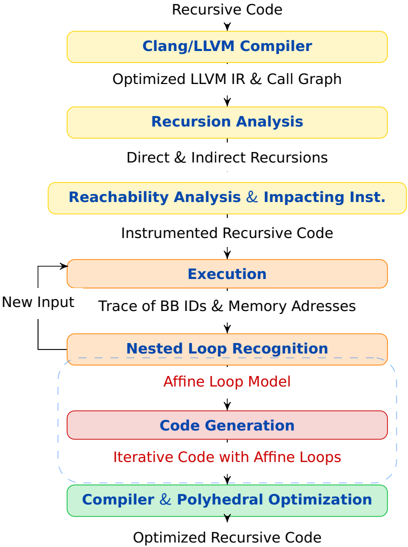
Nested Loop Reconginition (NLR) algorithm applications:

1. program behavior modeling for any measured quantity such as memory accesses
2. execution trace compressing
3. value prediction

(ketterlin & Clauss, GGO 2008)

Output: Affine Loop Model

Recursion to Affine Loop Nest Transformation

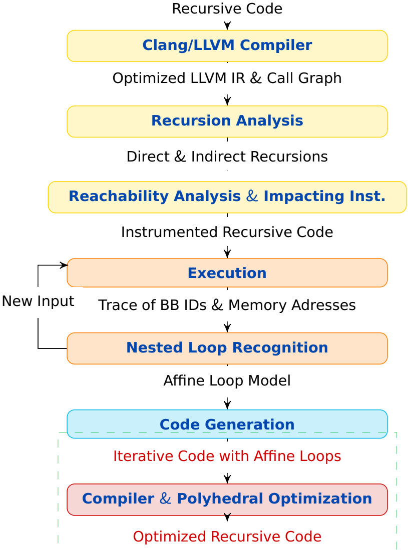


Input: Affine loop model

1. Extract NLR resulting loop nests structures
2. Construct loops in the LLVM IR using:
   - Instrumented basic blocks
   - Interpolated memory addresses

Output: Iterative code with affine loops

Polyhedral Optimizations



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Optimized Recursive Code

Input: Iterative code with affine loops

- Apply LLVM optimization passes
- Use polly LLVM polyhedral optimizer (Grosser et al., PPL 2012)

Output: Optimized recursive code

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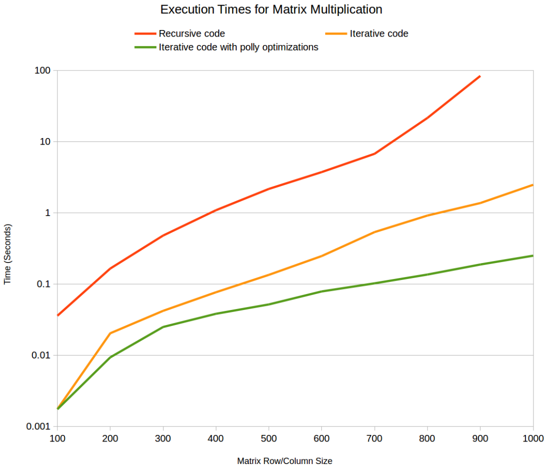
Recursive Matrix Multiplication

```
void MatrixMultiplication(int A[N][N], int B[N][N]){
 static int row=0, column=0, index=0;

   if (row >= N)
          return;

        if(column < N){
          if(index < N){
           C[row][column]+= A[row][index]*B[index][column];
                index++;
                MatrixMultiplication(A, B);
          }
          index=0;
          column++;
          MatrixMultiplication(A, B);
        }
        column=0;
        row++;
        MatrixMultiplication(A, B);
}
```

Recursive Matrix Multiplication Analysis Results

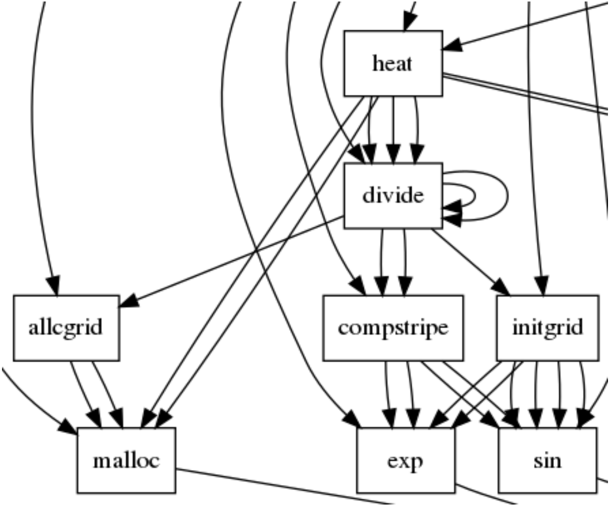
```
for i0 = 0 to N-1
 for i1 = 0 to N-1
  for i2 = 0 to N-1
   val MatrixMultiplication::if.then4 //IR basic block
   ...
   load // memory read
   val MEM1 + 4*N*i0 + 4*i2 //memory address in terms of loops indices
   ... //repetitive memory access patterns
   load
   val MEM2 + 4*i1 + 4*N*i2 //4 is the size of an integer
   ...
   val load
   val MEM3 + 4*N*i0 + 4*i1
   val store // memory write
   val MEM3 + 4*N*i0 + 4*i1
   ...
  val MatrixMultiplication::if.end15
  ...
 val MatrixMultiplication::if.end17
 ...
for i0 = 0 to N*N-1
 for i1 = 0 to N-1
  val MatrixMultiplication::if.end17
  ...
  val MatrixMultiplication::if.end15
  ...
 val MatrixMultiplication::if.end15
 ...
```

Recursive Matrix Multiplication Experimental Results



Serial execution (gcc -O3)

Heat - REAPAR Benchmarks



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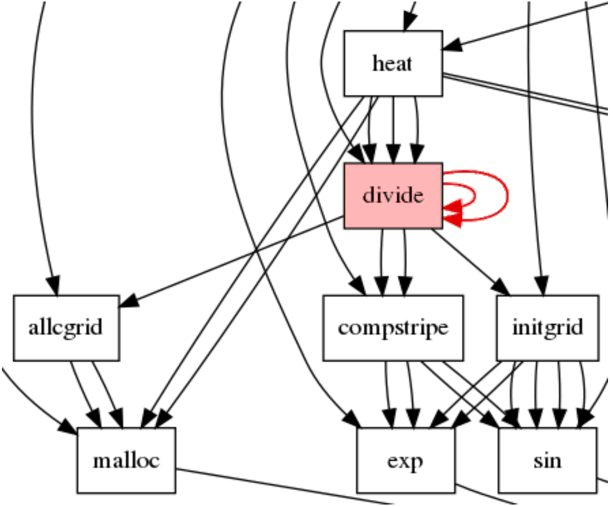

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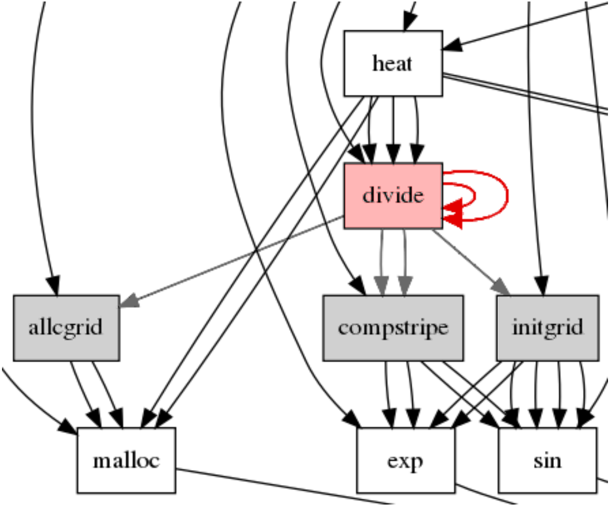

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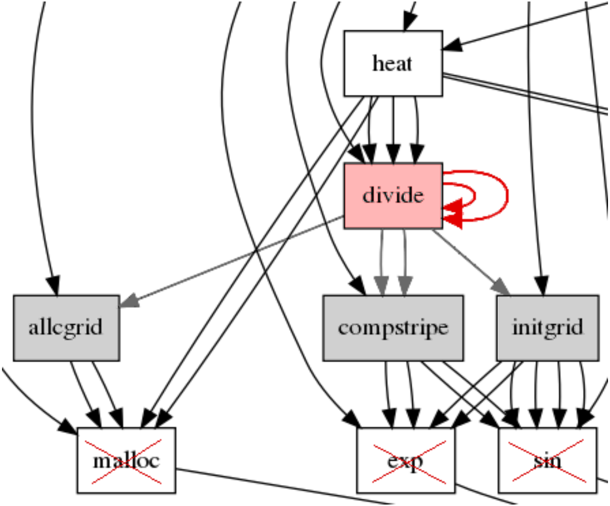


Heat

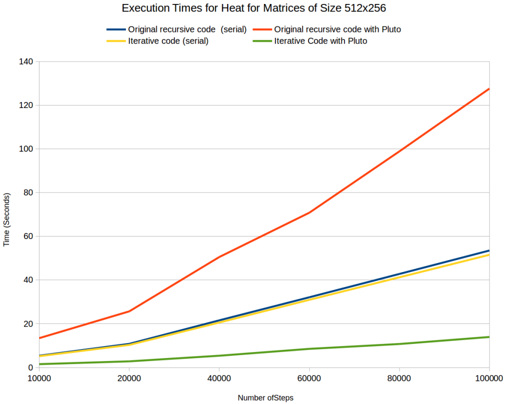
The function compstripe involves interesting linear loops

```
void compstripe(register double **new, register double **old, int lb, int ub)
{
  register int a, b, llb, lub;
  llb = (lb == 0) ? 1 : lb;
  lub = (ub == nx) ? nx - 1 : ub;
  for (a=llb; a < lub; a++) {
    for (b=1; b < ny-1; b++) {
      new[a][b] =  dtdxsq * (old[a+1][b] - 2 * old[a][b] + old[a-1][b])
              + dtdysq * (old[a][b+1] - 2 * old[a][b] + old[a][b-1])
               + old[a][b];
    }
  }
  for (a=llb; a < lub; a++)
    new[a][ny-1] = randb(xu + a * dx, t);
  for (a=llb; a < lub; a++)
    new[a][0] = randa(xu + a * dx, t);
  if (lb == 0) {
    for (b=0; b < ny; b++)
      new[0][b] = randc(yu + b * dy, t);
  }
  if (ub == nx) {
    for (b=0; b < ny; b++)
      new[nx-1][b] = randd(yu + b * dy, t);
  }
}
```

Heat Analysis Results

```
for i0 = 0 to Number_of_Steps-1
 for i1 = 0 to 14
  for i2 = 0 to 509
   val compstripe::for.body10, MEM1 + 8224*i1 + 8*i2, MEM2 + 8224*i1 + 8*i2, MEM3 + 8224*i1 + 8*i2
    , MEM4 + 8224*i1 + 8*i2 , MEM5 + 8224*i1 + 8*i2, MEM6 + 8224*i1 + 8*i2
 for i1 = 0 to 14
  val compstripe::for.body63, MEM7 + 8224*i1
 for i1 = 0 to 14
  val compstripe::for.body81, MEM8 + 8224*i1
 for i1 = 0 to 511
  val compstripe::for.body97, MEM9 + 8*i1
 for i1 = 0 to 61
  for i2 = 0 to 15
   for i3 = 0 to 509
    val compstripe::for.body10, MEM10 + 131584*i1 + 8224*i2 + 8*i3, MEM11 + 131584*i1 + 8224*i2 + 8*i3, MEM12 + 131584*i1 + 8224*i2 + 8*i3
     , MEM13 + 131584*i1 + 8224*i2 + 8*i3, MEM14 + 131584*i1 + 8224*i2 + 8*i3, MEM15 + 131584*i1 + 8224*i2 + 8*i3
  for i2 = 0 to 15
   val compstripe::for.body63 , MEM16 + 131584*i1 + 8224*i2
  for i2 = 0 to 15
   val compstripe::for.body81 , MEM17 + 131584*i1 + 8224*i2
 for i1 = 0 to 14
  for i2 = 0 to 509
   val compstripe::for.body10, MEM18 + 8224*i1 + 8*i2, MEM19 + 8224*i1 + 8*i2, MEM20 + 8224*i1 + 8*i2
    , MEM21 + 8224*i1 + 8*i2, MEM22 + 8224*i1 + 8*i2, MEM23 + 8224*i1 + 8*i2
 for i1 = 0 to 14
  val compstripe::for.body63 , MEM24 + 8224*i1
 for i1 = 0 to 14
  val compstripe::for.body81 , MEM25 + 8224*i1
 for i1 = 0 to 511
  val compstripe::for.body115 , MEM26 + 8*i1
   ...........
   ...........
   ...........
   ...........
```

Heat Experimental Results



The codes have been parallelized by Pluto using OpenMP 24 threads (AMD Opteron 6172 2x12-cores - gcc -O3 -fopenmp)

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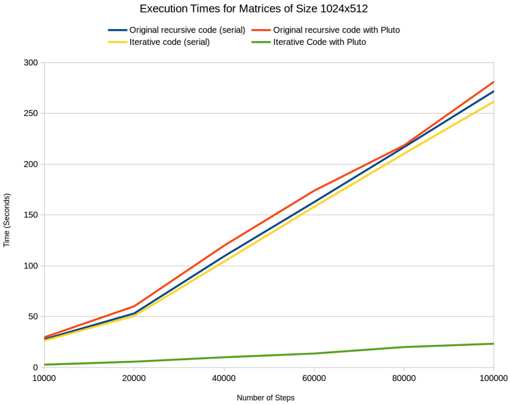

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Conclusion

A proof of concept for an automatic recursion-to-affine-loop transformation:

- involving static and dynamic analysis
- transformation passes
- polyhedral optimizers

Achievements

1. extends the polyhedral model applicability to non-loop control structures
2. brings the handled recursive functions to a higher level of optimizations

Future Works

Our future works include:

1. Performing dynamic analysis for recursive behavior at runtime
2. Inducing verification features to obtain a predictive model
3. Tackling input dependent recursive codes

Thank you

Questions ?